# Energy Consumption in Automatic Control Systems: An Empirical Study

S. Álvarez Rodríguez
Tecnológico Nacional de México: Instituto Tecnológico José Mario Molina Pasquel y Henríquez, Unidad Academica Lagos de Moreno, Jalisco, México
sergio.alvarez@lagos.tecmm.edu.mx

***Abstract*— This paper presents a comparative analysis of the energy consumption and performance of two widely used control strategies: Sliding Mode Control and Proportional-Integral-Derivative (PID) control. While SMC is known for its robustness to structural uncertainties and unmodeled dynamics, it also introduces the undesired effect of chattering. In contrast, PID control offers smoother, continuous control signals and is extensively applied across industrial systems. Adopting an energy-based physics perspective, this study focuses on evaluating the kinetic energy consumption of a controlled plant under each strategy and assesses their respective convergence behaviours with respect to a reference signal. The primary objective is to quantify the trade-off between robustness and energy efficiency to inform practical implementation decisions.**

***Keywords*— PID control, sliding modes, energy consumption.**

## I. Introduction

Automatic control systems can also be analyzed from an energy-based physics perspective to assess the efficiency and energy consumption of the underlying processes.

A variety of control laws can be employed depending on the specific system requirements. Broadly, controllers can be categorized into two types: those that generate continuous control signals and those that produce discontinuous signals. This paper investigates the energy-related behavior of both categories.

Sliding Mode Control (SMC) is a nonlinear control strategy grounded in the theory of dynamical systems. Its principal objective is to steer the system's state trajectory toward a predefined sliding manifold within the state space and to maintain the trajectory on this manifold for the remainder of the operation, thereby ensuring adherence to the desired system dynamics. This control methodology is particularly distinguished by its strong robustness with respect to structural uncertainties, unmodeled dynamics, and bounded external disturbances. To accomplish this, the controller employs a discontinuous control law that modifies the system's dynamics in real time, thereby enforcing the sliding mode behavior (see [1-3]).

The Proportional-Integral-Derivative (PID) control is a type of automatic controller widely used in control systems to regulate the output of a process, minimizing the error between the desired output and the actual output of the system. The PID controller adjusts the control signal through a combination of three terms (which are functions of the tracking error): proportional, integral, and derivative, each of which has a distinct impact on the system's behavior.

On the other hand, there are a multitude of ways to measure the energy consumed by a control system. For this work, the kinetic energy of the plant to be controlled will be taken into account.

### *A. Problem Statement*

Sliding mode control is widely recognized for its robustness to unmodeled disturbances and unmodeled dynamics; however, it also induces an undesirable chattering effect (see [4-7]). The objective of this study is to quantify the energy consumption of sliding mode controllers in comparison to that of continuous-signal PID controllers. Additionally, the study aims to assess the convergence rates of each controller relative to the reference signal, thereby providing valuable information to support implementation decisions.

### *B. Justification*

The novel approach in this study is particularly relevant due to the widespread use of automatic control systems in modern industry and their significant influence on both operational costs and environmental sustainability. Enhancing their energy consumption and efficiency could not only reduce operational expenses but also support compliance with environmental regulations and lower carbon emissions.

By investigating the energy-consumption vs robust control properties implications of some popular control systems, this study aims to open new avenues for the understanding of these systems from a thermodynamic perspective.

### *C. General Objective*

Quantify the energy consumption of variable gain sliding modes, with respect to the traditional PID controller, mentioning the general performance of each type of control.

### *D. Hypothesis*

Sliding modes consume a much higher amount of power than traditional PID control. This higher consumption translates into faster and more accurate convergence to the reference signal.

## II. Control System

To conduct this study, we consider an electromechanical system consisting of an electrically actuated kinematic pair "piston-cylinder" (where the piston moves inside the fixed cylinder) whose mathematical model is presented below in state-space form.

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -I^{-1}(0.27\, sgn(x_2 + 0.14\, x_2) + I^{-1}\, u, \qquad (1)$$

where, $x_1$ represents the position of the piston's actuator relative to the cylinder, $x_2$ is the velocity of the same actuator, $I^{-1}$ denotes the inverse of the inertia of the moving part of the mechanism, $u$ is the control signal produced by some controller, and evidently, $\dot{x}_1$ and $\dot{x}_2$ are the first time derivative of the position and velocity, respectively. The mathematical model does not include the force of gravity because it is considered to be in a horizontal position.

In control systems, the term "tracking error" refers to the difference between the desired output and the actual output over time, and it is commonly used to evaluate the performance of controllers, as it indicates how well the system is able to maintain the desired trajectory, especially in the presence of disturbances, noise, or modeling errors. The following equation computes the tracking error for every one of the controllers under study.

$$e = x_1 - x_{Ref}, \qquad (2)$$

where, $x_{Ref}$ is the reference signal.

The control signal $u$ in equation (1), can be obtained through a variety of methods, including classic controllers such as PID and robust controllers such as sliding modes. For this study, three different controllers are proposed: a sliding controller with variable gains PID+$\delta$ which generates $u_1$ in equation (3); a classical PID control which generates $u_2$ in equation (4); and also sliding controller with variable gains PID which generates $u_3$ in equation (5).

$$u_1 = [k_p|e| + k_i \int_{t0}^{t1}|e|dt + k_d\, d|e|/dt + \delta]\, sgn(e), \qquad (3)$$

$$u_2 = k_p\, e + k_i \int_{t0}^{t1} e\, dt + k_d\, de/dt\,, \qquad (4)$$

$$u_3 = [k_p|e| + k_i \int_{t0}^{t1}|e|dt + k_d\, d|e|/dt]\, sgn(e), \qquad (5)$$

where, |e| represents the absolute value of the tracking error, and $sgn(e)$ denotes the well-known sign function of the tracking error. $k_p$, $k_i$, and $k_d$, are the proportional, integral and derivative gains, respectively (see [8-11]).

## III. Simulation Results

By simulating the control system (1) in specialized software, results are obtained as that exemplified by the enlarged view of Fig. 1, where the cyan line represents the performance of controller (3), the black line is the performance of controller (4), the red line is for the control law (5), while the blue line represents the reference signal to track.

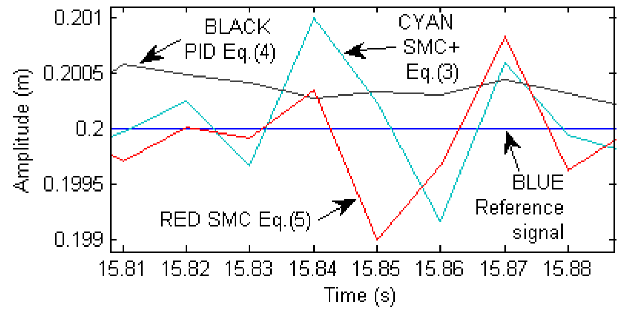


Fig. 1. Zoom view to show the colors notation: cyan, black and red for equations (3), (4) and (5), respectively, and blue for the reference signal.

For all the figures that follow, the color notation is the same than Fig. 1, and the parameters included in equations (1)—(5) are:

$I^{-1} = 5;\, \delta = 0.2;\, k_p = 500;\, k_i = 50;\, k_d = 10;\, \delta = 0.2,$

where, also external perturbations (shocks) are included with a period of 12 s and a pulse width (% of period) of 5, the amplitude of these disturbances is 0.2 m.

The reference signal $x_{Ref}$ is constituted by a square wave form with an amplitude of 0.2 m and a frequency equals 0.1 Hz.

Fig. 2, shows the tracking process of the mentioned reference signal, and where also the disturbances as external shocks can be observed, which are different from the overshoots coming from the control signal. In Fig. 2, only the red line is observed because all the other colors are below it.

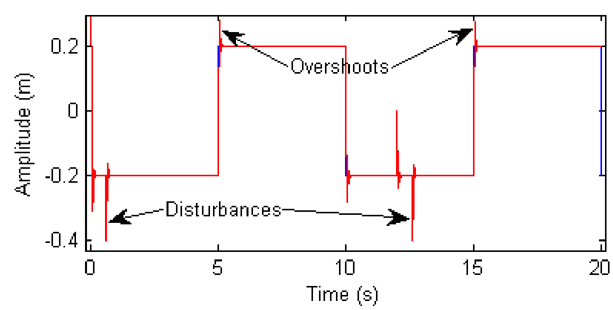


Fig. 2. Tracking of the square wave form as the Reference signal, with an amplitude of 0.2 m and a frequency of 0.1 Hz.

Fig. 3, shows the performance of the three controllers in the first negative half-cycle. As can be seen, sliding mode controllers are more robust than the simple PID, since PID does not reach convergence on the reference, however, sliding modes produce more chattering, which indicates that these latter controllers consume much larger amounts of energy. The PID is far from the reference an average distance of 0.825 mm,

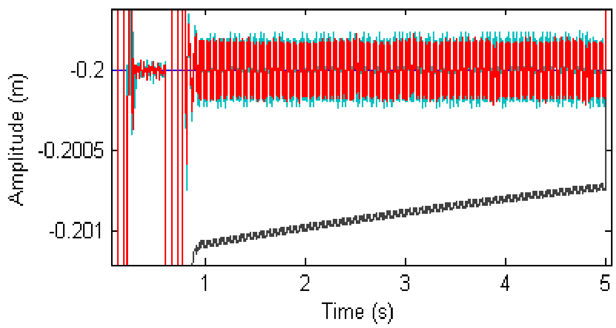


Fig. 3. Performance of controllers with a 1.5 thousandths (m) approach on the amplitude (vertical) axis.

Figure 4 is similar to Figure 3, but shifted to the third half-cycle with a different scale on the vertical axis, where the effect of the perturbations can be appreciated. In this perspective, neither the chattering nor the magnitude of the convergence of the controllers can be clearly observed.

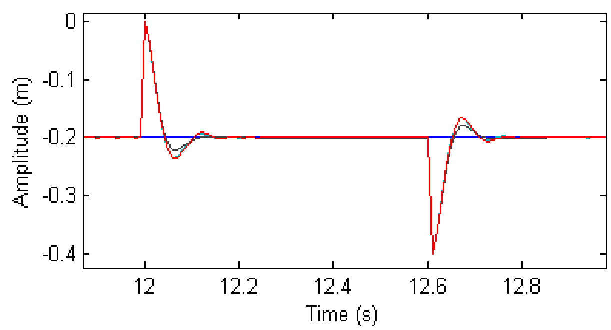


Fig. 4. Performance of controllers with a with an approximation of 4 tenths (m) on the amplitude axis.

Fig. 5 shows, as well as the previous one, the third half-cycle of the reference. However, it can be seen the details of the chattering that each controller produces because the vertical axis scale is magnified. Before the first disturbance, which occurs at 12 s, controller (5) produces much more chatter than controller (3), but between disturbances 1 and 2, this phenomenon is reversed. After disturbance 2, which occurs at 12.7 s, the chattering levels are similar for both controllers. It can also be seen that the PID produces minimal chattering and never reaches convergence on the reference signal.

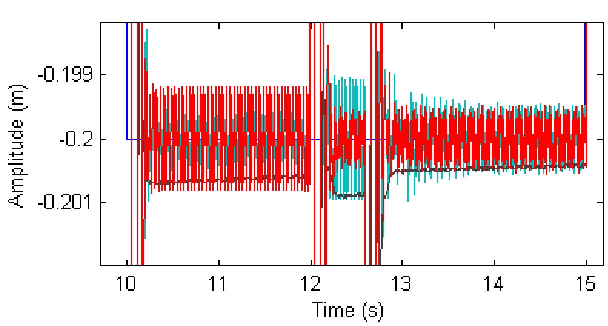


Fig. 5. The third half-cycle of the reference.

Sliding modes reach convergence almost instantaneously after each disturbance. The PID control falls off convergence, on average, 0.6 mm before the first disturbance, 0.85 mm between the first and second disturbances, and 0.5 mm after the second disturbance.

The above results have been obtained without noise being injected into the system, however, when the system is disturbed with a white noise power of $10^{-7}$ N, being the sampling time of 0.01 s, then, control signals become chaotic, as can be seen in Fig. 6. Note that both sliding modes maintain similar chattering, while the PID considerably increases its chattering level although, as in the previous results, it does not reach convergence either. The behavior of the sliding modes under these conditions indicates that they reach convergence quickly, and the level of chattering they produce is similar. On the other hand, the PID falls short of the reference by an average of 0.5 mm.

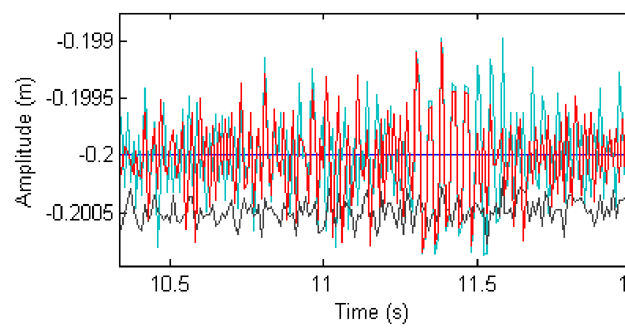


Fig. 6. Performance of controllers being subjected to noise.

Once these results have been obtained, the kinetic energy of each of the controllers will be obtained using the following equations

$$E_K = \frac{1}{2} m\, v^2, \tag{6}$$

with,

$$v = 2\pi f_c\, A, \tag{7}$$

where, $E_K$ is the kinetic energy, $m$ is the piston's mass, $v$ is the piston's velocity, $f_c$ is the chattering's frequency, and $A$ is the amplitude of the chattering (see [12-14]).

## IV. Dissipated Energy and Efficiency

The final figures illustrate representative examples of the control system's behavior under the specified conditions. To analyze the system more comprehensively, additional data was collected to determine the frequency and average amplitude of chattering. These results are summarized in Tables I and II, respectively, for each controller evaluated within the region highlighted in Fig. 5.

TABLE I
CHATTERING FREQUENCIES (HERTZ)

| **Controller** | **Location in Fig. 5** | | |
|---|---|---|---|
| | ***Before 1st perturbation*** | ***In between both perturbations*** | ***After 2nd perturbation*** |
| Eq. (3) | 50 Hz | 25 Hz | 50 Hz |
| Eq. (4) | 50 Hz | 50 Hz | 50 Hz |
| Eq. (5) | 25 Hz | 50 Hz | 50 z |

TABLE II
AVERAGE CHATTERING AMPLITUDES (METERS)

| **Controller** | **Location in Fig. 5** | | |
|---|---|---|---|
| | ***Before 1st perturbation*** | ***In between both perturbations*** | ***After 2nd perturbation*** |
| Eq. (3) | $8 \times 10^{-4}$ | $18 \times 10^{-4}$ | $11 \times 10^{-4}$ |
| Eq. (4) | $3.5 \times 10^{-5}$ | $3.5 \times 10^{-5}$ | $3.5 \times 10^{-5}$ |
| Eq. (5) | $16 \times 10^{-4}$ | $7.5 \times 10^{-4}$ | $8.5 \times 10^{-4}$ |

Taking the piston's mass $m = 0.2$, Table III is obtained with equations (6)—(7) and data from Tables I and II.

TABLE III
AVERAGE KINETIC ENERGIES (JOULES)

| **Controller** | **Location in Fig. 5** | | |
|---|---|---|---|
| | ***Before 1st perturbation*** | ***In between both perturbations*** | ***After 2nd perturbation*** |
| Eq. (3) | $2.5 \times 10^{-2}$ | $2.83 \times 10^{-2}$ | $3.5 \times 10^{-2}$ |
| Eq. (4) | $0.1 \times 10^{-2}$ | $0.1 \times 10^{-2}$ | $0.1 \times 10^{-2}$ |
| Eq. (5) | $2.5 \times 10^{-2}$ | $2.36 \times 10^{-2}$ | $2.7 \times 10^{-2}$ |

Table III, offers the energy consumption results of the controllers studied, giving rise to the following conclusions.

## V. CONCLUSION

Sliding Mode controllers dissipate almost thirty times more energy than traditional PID control. In exchange for this enormous energy consumption, it achieves enormous robustness against disturbances working with demanding reference signals. The PID is energy efficient but not very robust in tracking the reference. The sliding mode with an additional term is energetically a little less efficient than the one with a simple PID gain, but both maintain a similar robustness, which leads to the conclusion that the additional term $\delta$ does not have a decisive positive influence (please see [8]).

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